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THE PRESSURE BUMP INSTABILITY IN THE FERMILAB DOUBLER

The major limitation to luminosity in the CERN ISR is the existance of the phenomenon called the pressure bump instability. The circulating beam ionizes residual gas molecules, and because of the positive potential of the beam relative to the wall, the ions are accelerated to the wall. When they hit the wall, the ions knock gas molecules off of the wall, which may in turn be ionized by the beam. If the beam current is high enough, and if a large number of molecules are desorbed per incident ion, this can result in a divergent pressure which will eventually destroy the beam.

An easy way to see this, is to estimate the equilibrium pressure. Without the beam, the gas load is Q. For a pumping speed S, the pressure is

$$P_{equil} = \frac{Q}{S}.$$

The desorption of molecules from the wall is proportional to the beam current (possibly to some power other than one) and also proportional to the equilibrium pressure. We define the parameter λ, the desorption coefficient,

$$\eta = \frac{\text{molecules out - ions in}}{\text{ions in}},$$

and a proportionality constant k, which contains the ionization cross section, etc. Then, with the beam on,

$$P_{equil} = \frac{Q=k\eta IP_{equ}}{S}.$$

Hence,

$$P_{equil} = \frac{Q}{S-k\eta I}.$$

The equilibrium pressure becomes infinite for

$$I_{crit} = \frac{S}{k\eta}.$$

It is clear that for a warm bore machine, either S must be very large, or η very small. At the ISR, both of these conditions apply, and currents of 30 amperes have been reached.

In a cold bore machine, the criterion is

$$I < I_{crit} = \frac{\pi r}{2} \frac{e\bar{v}s}{\sigma\eta},$$

where $\bar{v}$ is the average velocity of the molecules coming off the wall, r is the beam pipe radius, s is the sticking probability for the molecules, and σ is the ionization cross section.

It was discovered in 1972 by Erents and McCraken[1] that the desorption coefficient of condensed gases at low temperature is very large. At the ISR, $\eta \approx 1$ or 2, or even negative. For thick films of a few monolayers at liquid helium temperature, η can be greater than 50,000.[2] Compensating for this is the very high wall pumping speed in a cold bore, which does not allow the desorbed molecules to bounce around until they are ionized by the beam.

In order to make an estimate of the critical current, we must estimate the various parameters in the above equation.

$\bar{v}$ - the average speed of the desorbed molecules. A lower limit is probably the thermal velocity at 4.5 k, about 1.8×10^4 cm/sec. A reasonable upper limit would be the order of a few tenths of an electron volt, typical sputtering energies.

η - desorption coefficient. For ion energies below 1 keV, and for coverages less than about 10 monolayers, it appears that η depends linearly on the incident ion energy, and linearly on the coverage.[2] Since we cannot tolerate coverages greater than a few monolayers of helium in any case, we will use, for η

$$\eta \approx 10^5 \cdot \varepsilon_I(\text{keV}) \cdot \theta \text{ (monolayers)},$$

which appears to roughly agree with the data.

$\mathcal{E}_I$ - ion energy. The potential of the beam, relative to the beam tube is

$$V = \frac{BI}{2\mathcal{E}_0(f2R)} \ln \frac{r_o}{r_b} = 2000 \times I$$

where B = bunching factor ≈ 10

f = revolution frequency

R = machine radius

r_c = beam tube radius = 38 mm

r_b = beam radius = σ(95%)

for ε = .01 > mm-mrad

and β = 70 meters.

s - sticking probability. For coverages of a few monolayers, the molecular energy is sufficiently low so that the sticking probability of all molecules, including helium is one.

σ - ionization cross section. An extension of measurements made at CERN[3] indicate that the ionization cross section for helium will be less than $.5 \times 10^{-18}$ cm^2.

Plugging these in, for thermal velocities, we get

$$\Theta I_{crit}^2 \cong 17.$$

For 3 monolayers, that corresponds to 2.4 amps. The Main Ring, at present, has a peak current of .16 amps. For more reasonable coverages of say .01 monolayers, I_{crit} = 40 amps.

Experiment

A cold bore section has been run at the ISR.[4] Although the results are somewhat confusing, the conclusion is that they have been able to run up to 14 amperes successfully. This is particularly interesting, since the cold bore

region had pressures that were 10 times the average pressure in the ISR, with very large coverages. This reference is included for information.

Our conclusion is that we will not have any problem with ion induced desorption in the Energy Doubler.

References

1. "Desorption of Solid Hydrogen by Energetic Proton, Deuterons, and Electrons;" S. K. Erents and G. M. McCracken, UKAEA Research Group, Culham Laboratory, CLM-P 330.
2. "Ion Desorption of Condensed Gases;" N. Hilleret and R. Calder; CERN-ISR-VA/77-33 (May 12, 1977).
3. "Basic Design of the Vacuum System for 400 GeV LSR with Normal Magnets;" D. Blechschmidt; CERN-ISR-VA/75-29 (July 4, 1975).
4. "A Vacuum Cold Bore Test Section at the CERN ISR;" C. Benvenuti, R. Calder, and N. Hilleret; CERN-ISR-VA/77-19 (March 1977).

APPENDIX

In order to understand the phenomena in a vacuum system, it is profitable to introduce some formalism. We start with the diffusion equation

$$\vec{j} = \vec{\nabla} k\, n,$$

where n is the density of gas molecules inside the volume V, enclosed by the surface $\vec{s}$, k is a diffusion constant with units of area/time, and $\vec{j}$ is the net current density of molecules leaving the volume. From conservation of matter, the total number of molecules leaving the volume is

$$\oint \vec{j} \cdot d\vec{s} = \text{sources inside V - time rate of change of density inside V.} \qquad (1)$$

There are three sources:

1. Leaks. The total leak rate is ΣU_i, where U_i is the leak rate of the i^{th} leak, in molecules/sec, or more conveniently torr-liters/second (1 torr liter = 3.5×10^{19} molecules at 300 K). The position dependence is obviously a delta function for each leak.
2. Wall outgassing. This includes the normal gas evolution of materials. Stainless steel evolves mostly hydrogen. If it is vacuum baked at 950°C, and otherwise treated in fancy ways, the outgassing rate at room temperature can be $\lesssim 10^{-12}$ torr liters/cm^2-sec. At liquid helium temperature the gas evolution of stainless steel is so small as to be negligible.
3. Transition from surface phase to gas phase. If the surface of a material is covered by θ molecules/cm^2, then some of those molecules will leave the surface and enter the gas phase. This is not to be confused with outgassing, which involves gas which is a constituent of

the material. At room temperature, the surface phase evolution of gas from clean stainless steel is $\frac{d\theta}{dt} \approx 10^{-5}/t(\text{sec})$ torr liters/cm^2-sec, and includes water, CO, CO_2, N_2 and other gases. At liquid helium temperatures, this phenomenon is also very small, except for helium itself, and hydrogen. However, effects due to circulating beam can make this effect very large, if θ is large.

Returning to equation 1:

$$\oint \vec{j}\, d\vec{s} = \int \Sigma U_i\, (\vec{x}_i - \vec{x}_i)dV + \int q dV - \int \frac{\partial \theta}{\partial t} dV - \int \frac{\partial n}{\partial t} dV. \tag{2}$$

From Gauss's Law

$$\oint \vec{j} \cdot d\vec{s} = \int \vec{\nabla} \cdot \vec{j}\, dV. \tag{3}$$

For a pipe of uniform cross sectional area A and perimeter w

$$dV = A\, dx. \tag{4}$$

In addition, the outgassing and phase transition occurs only at the walls, and is zero otherwise, so it is convenient to use parameters that are represented per unit length. Then it is particularly easy to make this into a one-dimensional problem

$$qdV = qw\, dx = a\, dx \tag{5}$$

and

$$\frac{\partial \theta}{\partial t} dV = w \frac{\partial \theta}{\partial t} dx \tag{6}$$

and

$$U_i\, \delta(\vec{x}_i - \vec{x}_i) dV = U_i\, \delta(x_i - x_i')\, dx. \tag{7}$$

For simplicity, we will ignore the leaks, and we have

$$A \int \frac{\partial j_x}{\partial x} dx = \int a\, dx - \int w \frac{\partial \theta}{\partial t} dx - A \int \frac{\partial n}{\partial t} dx. \tag{8}$$

Substituting into the diffusion equation

$$j_x = -k\frac{\partial n}{\partial x}, \tag{9}$$

$$A\frac{\partial n}{\partial t} = Ak\frac{\partial^2 n}{\partial x^2} + a - w\frac{\partial \Theta}{\partial t}. \tag{10}$$

Ak is the specific conductivity - the conductivity of a tube of unit length, and is called c. In the molecular flow region, that is, where collisions with the walls occur more frequently than collisions with other molecules

$$c = \frac{4}{3}\bar{v}\frac{A^2}{w}, \tag{11}$$

where $\bar{v}$ is the average velocity of molecules = $(\frac{8R_oT}{\pi M})^{1/2} \cong 1.4 \times 10^4\sqrt{T/M}$.

The phase transition term involves the previously mentioned part that is inversely proportional to time, a term which depends on gas sticking to the wall, and beam induced removal from the wall.

If the molecules have an average speed $\bar{v}$, then the number hitting the wall per second is $n\bar{v}/4$, and the number which sticks is

$$\frac{n\bar{v}}{4}s \qquad 0 \leq s \leq 1.$$

At cryogenic temperatures, and for $\bar{v}$ less then an electron volt, $s \approx 1$.

The desorption part could involve terms due to beam scraping, etc., but we will only concern outselves with ion desorption. If the beam current is I, then the number of particles passing a point in the ring in a second is I/e. The number of ions formed is then

$$\frac{I}{e}n\sigma,$$

where σ is the ionization cross section.

The ionization cross section[1] is about $0.5 \times 10^{-18} cm^2$ for hydrogen or helium. The number of ions hitting the wall per unit area is then

$$\frac{In\sigma}{ew},$$

and each ion will knock out some number of gas molecules. We define the

desorption coefficient

$$\eta = \frac{\text{Molecules out - ions in}}{\text{Ions in}}$$

where η is a function of the ion energy, wall coverage, gas type, ion type, temperature, and probably everything else. The final result is

$$\frac{\partial \Theta}{\partial t} = \left(\frac{\bar{\nu} s}{4} - \frac{\sigma}{ew}\eta I\right) n - \frac{\Theta}{T}, \tag{12}$$

where T is a time constant, and

$$A\frac{\partial n}{\partial t} = a + c\frac{\partial^2 n}{\partial x^2} - w\frac{\partial \Theta}{\partial t}. \tag{13}$$

The boundary conditions for a warm system are gotten in the following way:

If we put pumps at ±L, each with pumping speed s, then the gas lead that each pump handles is

$$Q = Sn. \tag{14}$$

The boundary condition is that each pump must handle all the gas that enters it. That is

$$Aj_x \mp \frac{1}{2} Q = 0 \text{ at } x = \pm L. \tag{15}$$

The 1/2 is due to the assumption that the pump receives gas from both directions. Then

$$Ak\frac{\partial n}{\partial x} \pm \frac{1}{2} Sn = 0,$$

or

$$c\frac{\partial n}{\partial x} \pm \frac{1}{2} Sn = 0 \text{ at } x = \pm L. \tag{16}$$

The solution of equation 13 with $\frac{\partial \Theta}{\partial t} = 0$ at equilibrium ($\frac{\partial n}{\partial t} = 0$) is

$$n_{(x)} = \frac{a}{2c}(L^2 - x^2) + \frac{2aL}{S}, \tag{17}$$

which gives the familiar parabolic gas distribution. The solution, with no outgassing, but with a leak U at $x = x_o$ is

$$n_{(x)} = \frac{2U}{s} + \frac{U}{c}(L + X_o - x), \tag{18}$$

giving a triangular distribution.

The solution of the actual differential equation is ridiculous, but fortunately, for our case, we can still find a solution for the critical current, I_c. In the cold bore case, all the terms are small, except for the terms within the parentheses in equation 12. The condition for no pressure bump instability is

$$I<I_{crit} = \frac{\pi r}{2}\frac{e\bar{v}s}{\sigma\eta}, \tag{19}$$

where $\bar{v}$ is the average speed of molecules knocked off of the wall, and r is the radius of the beam pipe.

For a warm bore system, a close approximation[3] is

$$I<I_{crit} = \frac{\pi^2 ec}{\eta\sigma(L+4\frac{c}{s})2}, \tag{20}$$

indicating that η must be small and the conductance and pumping speed must be large to have high critical currents.